SYNOPSIS

Title :" A STUDY OF SOME PROBLEMS ON BANACH SPACE THEOREMS FOR MAPPINGS ON DIFFERENT SPACES"

(a) Introduction

We propose to examine functional analysis as the study of function spaces and the linear operators derived by W. Rudin. These function spaces are endowed with the structure of a Banach space and the theory focuses on bounded linear operators between Banach spaces or normed vector spaces emphasizing the specific properties of the underlying function spaces. We propose to investigate its properties within Banach spaces, particularly in relation to isometries and applications to the theory of random processes. W. Linde introduced the measures in Banach spaces and extend this formulation to encompass Gaussian and Gleason measures. We also intend to critically analyze stochastic formulations and their integrability conditions. The study of stochastic structures offers a broad range of applications in the physical sciences. we extend comparative analysis of the Wiener integral and the stochastic integral with particular emphasis on the martingale approach.

(b) Work plan

E. Wong discussed the role of stochastic processes in dynamical systems, which forms part of our motivation. The notion of a normed ring-later recognized as a Banach algebra was introduced by M. Gelfand and W. Ambrose subsequently extend the term Banach algebra while studying certain structural theorems. Banach algebra theory is elegant, particularly in its use of commutative Banach algebras for representation theory. The Banach fixed point theorem as the contraction mapping theorem, states that every contraction on a complete metric space has a unique fixed point. We propose to reformulate Banach's fixed point theorems in finite-dimensional settings, with an emphasis on applications to programming semantics in theoretical computer science.

We extend the Banach spaces applications in both linear and nonlinear functions, operator theory, real and complex algebras, linear operators and their isomorphisms, as well as compact algebras. It plays a key role in the Banach contraction principle, fixed-point approximation, amenability theory and various analyses involving wavelength and

related properties of Banach spaces. We establish properties of module extension dual Banach spaces and examine the relationship between weak*-continuous homomorphisms of dual Banach spaces and Connes-amenability by B. E. Johnson. J. Ringrose extended the notion of amenability from von Neumann algebras to general Banach spaces by incorporating the dual space structure inherent in von Neumann algebras. we develop the concept of amenability for dual Banach algebras as formulated by V. Runde.

(c) Details of the project with chapterisation

We describe the essential features of vector spaces, including their dimensionality and applications. We develop the underlying structures of these spaces and the operators defined on them, noting that complex vector spaces differ in important ways from the more familiar real vector spaces. We formulate certain linear equations using suitable linear operators acting on Banach spaces. The basic properties of deformations and their relationship to linear operators on Banach and Hilbert spaces are also developed. We establish that a linear operator is continuous if it is bounded, introduce the norm of a bounded linear operator, and discuss its properties. Moreover, we derive unbounded linear operators, which play an important role in applications such as differential operators within the framework of Hilbert spaces. The Hahn–Banach theorem concerns bounded linear functionals defined on a subspace of a Banach space X, asserting that each such functional may be extended to a bounded linear functional on the entire space X. This result remains more general context, where X is any real vector space and the notion of boundedness is replaced by a bound determined by a given quasi-seminorm on X. We develop two notions of amenability—(i) Connes-amenability and (ii) strong Connes-amenability-both of which incorporate the weak*-topology on dual Banach algebras. The concept of amenability, introduced by B. E. Johnson, has long been regarded as a major obstacle in establishing a comprehensive theory for dual Banach algebras. However, as demonstrated by G. Gorach, adopting the weak*-topology provides the most suitable framework for formulating amenability in this setting. The whole proposed research project work is divided into four different Chapters are as follows.

Chapter - 1 Characterization of Banach Space and its impact on Bounded Linear Operators

The concept of a Banach space, one of the most fundamental notions in functional analysis is that of a bounded linear operator. The linear operator and linear map are equivalent-both referring to a map between vector spaces that preserves addition and scalar multiplication-the former is more commonly used, especially because many normed spaces encountered in applications are function spaces. The domain and codomain of a linear operator are normed vector spaces, it is natural to require continuity with respect to their norm topologies.

Chapter - 2 Comperative structure of Metric Spaces and Compact Sets

We derive the fundamental definitions of metric spaces and Banach spaces, accompanied by many important theorems. The discussion then turns to compact subsets of metric spaces, demonstrating that sequential compactness is equivalent to the requirement that every open cover admits a finite subcover.

Chapter - 3 Unbounded Operators on Hilbert Spaces and its impact

This chapter, we devoted to the spectral theory of unbounded operators on a Banach space X. The domain of an unbounded operator is a linear subspace $\mathrm{dom}(A) \subset X$, which in most important cases is dense, and the operator $A: \mathrm{dom}(A) \to X$ possesses a closed graph. The chapter introduces the adjoint of an unbounded operator and derive the spectra of unbounded normal and self-adjoint operators.

Chapter - 4 Application of Finite-Dimensional Banach Spaces

we examine finite-dimensional normed vector spaces, highlighting the properties that set them apart from infinite-dimensional spaces, which drive the core of functional analysis. Finite-dimensional normed vector spaces are complete, all of their linear subspaces are closed, every linear functional on them is continuous, and their closed unit balls are compact.

(d) Hypothesis

Banach spaces have extensive applications in linear and nonlinear function theory, operator theory, real and complex algebras, linear operators and their isomorphisms, compact algebras. They form complete normed vector spaces, as described by H. G.

Dales and F. Edward Felly. We establish here the hereditary properties associated with the existence of Gelfand theory. We consider Banach spaces that exhibit the features of a commutative Gelfand theory. While the Gelfand theory of a commutative Banach algebra can be characterized by a set of axioms, the corresponding theories for arbitrary Banach algebras are more complex and require broader formulations.

(e) Methodology

The research project to be submitted is purely theoretical in its nature. The relevant books related to the preparation for research project are available in the central library, B. N. M. University. The assistance of consultation in National Library (Science Section) Calcutta and Indian Statistical Institute may be availed for recent Books and Journals on the topic of the research.

(f) References

1.	E. R. Lorch, (1939)	Means of iterated transformations in reflexive vector spaces, Bull. Amer. Math. Soc. 45, 945-947.
2.	R. C. James (1972)	Super reflexive Banach space, canad. J. Math 24 (896 - 904)
3.	W. Runin (1973)	Functional Ananysis Mc-Graw Hill
4.	J. Lindenstrauss and L. Tzafriri, (1977)	Classical Banach Spaces I, Springer-Verlag, Berlin, Sequence spaces.
5.	K. Yosida, (1980)	Functional Analysis, Sixth Edition. Grundlehren der Mathematischen Wissenschaften 123, Springer Verlag.
6.	F. Riesz, B. Sz.-Nagy, (1990)	Functional Analysis, Dover, New York.
7.	J. Lendenstrauss & V. D. Milman (1993)	The local theory of normed spaces and its applications to convexity, Hand book of Convex geometry Elsvier, (1150-1220)
8.	Robert E. Megginson, (1998)	An Introduction to Banach Space Theory. Graduate Texts in Mathematics 183, Springer,.
9.	MD. G.H. Dales (2000)	Banach algebras and automatic continuity, Clearndon Press, Oxford

10.	F. AL BIAC, N.J. KALTON (2006)	Topics in Banach space theory Springer, New York
11.	F. Albiac, N. Kalton, (2016)	Topics in Banach Space Theory, 2nd edition, Springer, Switzerland
12.	W. Brian, C. Stuart, (2024)	Small-dimensional Vector normed spaces, to appear in Israel J. Math.,

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